

A SHADOWGRAPH STUDY OF INTERMITTENT FLOW

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A THESIS

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NOTATIONS

x, y, z	linear distances; coordinate axes
a	sound velocity, atmospheric conditions
	density, atmospheric conditions
u	air velocity
p	pressure
k	$\frac{C_p}{C_v} \quad 1.4$
r	$\frac{1}{2} \left(\frac{2}{k-1} a + u \right)$
s	$\frac{1}{2} \left(\frac{2}{k-1} a - u \right)$
w	actual sound velocity, $a + u$ or $a - u$
b	distance, spark to jet axis
C	distance jet axis to film plate
d	distance that the film strip is above or below the jet axis
l	distance recorded on film strip

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FOREWORD

Purpose

The purpose of this thesis problem was to construct an apparatus for taking shadowgraphs, calibrate the apparatus and make a shadow-graph analysis of the exhaust of a small pulse jet.

Need for Information

With the advent of each fundamentally new internal combustion engine there arises a need for an understanding of the fundamentals which make the engine "new". A basis on which predictions may be made that indicates the optimum obtainable performance is needed and approaches mapped concerning the most expedient means of obtaining this optimum. The well developed steady flow relationships employed in the gas turbine or the much studied expansion in cylinders avail us little or nothing in studying this new field.

Intermittent flow in its simplest forms has never attracted by its knotty complexity too much affection in the breast of the engineer and the dearth of useable experimentation such neglect entails has left the ever willing mathematician very little wherewithall to juggle.

The pulsating jet engine, completely involved in intermittent flow, gives promise for such a rosy future that a through experimental coverage is deemed worthwhile.

CONCLUSIONS

The interface between the energy and momentum media is not a plane of discontinuity, but rather a thick ill defined gradual change in pressure and density. Since the turbulent nature of the insucked fresh "energy" charge has been encouraged as an aid to combustion, it is likely that this turbulence performs still another function, encouraging diffusion between the two media.

Sufficient air is sucked in the exhaust end of the tube at a high velocity to make the redesign of the tube's exhaust seem desirable. The intermittent flow emanating from the pulse jet becomes dampened a distance of 4 to 5 feet aft of the jet exhaust.

RECOMMENDATIONS

Future Study

Shadowgraph, or better, interferometer study of a glass walled pulse jet and its exhaust and intake as a function of time would greatly facilitate the improvement of the engine. Using a pressure pickup and a delay multivibrator to time the spark, this could be realized. To trigger the spark at a controlled time a short spark gap could be used in conjunction with the main spark. The main spark gap would be spaced just beyond the discharge point for the voltage used. The small gap would be placed opposite one of the electrodes and activated by the amplified pulse from the delay multivibrator.

A method for gaining quantitative analysis from shadowgraph photographs would be to pass through the jet stream a photographable sound wave of very small amplitude whose distortions would reveal the density, velocity variations incurred in the air stream. A similar approach would be to pass through the stream to be examined a relatively small supersonic jet stream whose deflection would again perform the required function.

Pulse Jet Design

Since the momentum of the jet exhaust as well as the precompression of a new charge are both benefitted by the air sucked in the exhaust of the jet, an obvious change suggested by this experiment would be to so fair the exhaust tube's exterior that the flow from all directions would enter with the minimum resistance.

Further augmentation could be obtained by putting one way check valves in the tube's exhaust walls to allow the insucked air to enter, but restrict the out flow to the geometry of the tube. Since a bent tube jet is operable, these valves could be so placed that in flight they would receive full benefit of the ram pressure by placing them on a portion of the tube, normal to the air flow.

TEXT

Introduction

Definition. The engine, treated in this report as the "pulse jet," has been called variously "reso-jet," "aeroresonator," "V-1 engine," or "buzz engine."

In its simplest form it consists of a straight tube, open on one end, intermittently closed on the other by one-way check valves. Air and fuel are introduced in the portion of the tube after the valves and ignited by use of a spark. As the products of combustion expand the check valves are closed and the gas is accelerated rearward through the open end of the tube. The momentum of the exhausting charge reduces the pressure inside the tube causing the valves to open admitting a fresh charge. This charge ignites automatically on contact with the residual hot products of the preceding combustion (the spark is needed for starting only) and the cycle repeats ad expirum or explosium.

Fuel is sprayed either before or after the valves.

History. Though the idea of the pulse jet must be nigh as old as the pulse, no record is known of experimentation until the twentieth century.

In 1908 Karavodine used a device similar to the pulse jet with the open end directed at turbine blades. His gadget produced 2 HP, an alarming 11 lbs/HP Hr (Benzene) fuel consumption, a loss of interest.

In Germany, Paul Schmidt, a consulting engineer and inventor, began a spare time assault on the pulse jet in 1928, received a patent in 1931, and a commission from the Air Ministry in 1935, at which time he employed

a small staff and his entire energies on the development of his gadget. Later the Air Ministry, alarmed mildly by Schmidt's failure to produce, instructed Professor Adolph Buseman to investigate the progress of Schmidt's research. Buseman concluded that the study was advantageous in the interest of theoretical investigation of non-steady flow alone, and, should Schmidt's or his, Buseman's, less spectacular predictions be realized, could be a useful device as a jet assistance for take off or for an emergency source of power at high speeds. Schmidt, therefore, was allowed to continue his work. In 1942 Argus was commissioned to produce the pulse jet and developed it in the form incorporated in the V-1 engine.

The German work to date is covered as well as censorship and governmental procurement permits by the references here included.

The German work, which reminds us of our own harried war time research, was a study in arbitrarily applied guesses in an effort to force quick results rather than to further the fundamental knowledge of the engine.

No information concerning pre V-1 era pulse jet study in countries other than Germany is available save an inconsequential bit of United States research.

E. B. Meyer played with a single pulse detonation engine in 1943; Dr. F. Zwicky of Aerojet Engineering Corporation considered an aeropulse device similar to Schmidt's.

Supported by the U. S. Navy Lt. W. Shubert constructed an aeropulse in 1944 which was run successfully, and, oddly enough, employed a tuned restricted pipe in place of the check valves employed by Schmidt.

After the capture of German V-1 engines all sorts of enthusiastic, if disorganized, research sprang up in this country and in England.

A memorandum report dated April 9, 1945, from AAF Materiel Command directed to all units Army, Navy and civilian engaged in intermittent jet study pretty well outlines the channels of investigation followed in the program. Herewith are the items dealing with the engine performance experimentation on which action was requested and the work, if any, which was done (as outlined by Mr. Valerino, N.A.C.A. Engine Lab, Cleveland).

"(A) Outline items of investigation in general development pulse jet.

"(B) Data

IJ-15-1 Engine, diameter 22", length 12', weight 300 lbs, thrust 900 lbs at 450 mph at 3 lbs fuel per lb thrust per hour compared to 1350 HP engine (80% prop. eff. and specific fuel consumption 2 lbs fuel per lb thrust per hour.

Max. pressure, 19 psi (gage) or compression ratio 2.3.

If instantaneous burning took place compression would approach 10 to 1.

Pipe itself weighs 260 lbs with metal stress 180 lbs psi.

"Valves

- (a) Leaf spring type - improvements on seating, tapering, modifying, etc."

Valves

- (a) Thin valves of lower frequency were tried, jet failed to resonate; tapered valves of approximately same frequency of German design were tried, jet resonated, valves lasted 30% (approx.) longer; life of original blades increased several 100% by Neoprene coating original valve seats.

- | | |
|--|--|
| "(b) Coil Springs" | (b) No record of experimentation. |
| "(c) Plate valves, spring loaded" | (c) Turner (N.A.C.A., Cleveland) loaded plates, approximately the same frequency as German blades, arranged in pairs on large seating, three pairs staggered and coupled to form complete grid; jet resonated and valve function photographed; not as efficient as German jet; reason thought that vibrations set up in blades bouncing on closing caused flow of air at undesirable times and caused a certain amount of blow back. |
| "(d) Resonant valve, transverse or rotary oscillation" | (d) Resonant valves, spring loaded to vibrate at frequency of jet, tried and jet failed to resonate; no record of transverse or rotary oscillation attempts. |
| "(e) Eccentric disc valve" | (e) No record of experimentation |
| "(f) Fixed inlet opening" | (f) Shubert's jet resonated but efficiency was low. |
| "Combustion" | Combustion |
| "(a) Combustion from spark rather than residual fuel" | (a) Impossible to purge residual flame. |
| "(b) Intermittent fuel injection" | (b) Attempts on part of Giannini & Co. to inject fuel in front of valves, successful, more efficient. Turner, N.A.C.A. had tried electronics control for fuel injection. He installed P.E. cell forward of valves which opened fuel feed as valves opened without appreciable increase in efficiency. |
| "(c) Cascading" | (c) Attempts to cascade combustion unsuccessful, engine would not resonate. |

- | | |
|--|---|
| "(d) Auxiliary combustion chamber ignition" | (d) No experimentation. |
| "(e) Carborated inlet (have fuel injected in venturi ahead of valves)" | (e) Multiple fuel injection in diffuser forward of blades normal to air stream of 200 mph gave reasonable atomization. |
| "(f) Ignition from hot spots and flame holders" | (f) Ignition ordinarily takes place from residual flame which remains in boundary area after the venturis; no record of flame holders employed. |
| "(g) Single explosion engines" | (g) Tried but did not prove too useful a tool in study. |
| "(h) Closed and tube engines (no valves)" | (h) Tried and resonance obtained but very small thrust. |
| "Construction and Arrangement" | Construction and Arrangement |
| "(a) Lighter material" | (a) Stainless steel employed, less weight; Giannini used magnesium tube and carborated on MiniJet weighing 4 oz. giving 2.5 lb thrust. |
| "(b) Shorter units" | (b) Various tube lengths were tried using original German grid and fuel consumption went up, thrust went down for any large deviation from designed length. |
| "(c) Common inlet and/or outlet" | (c) Common inlet tried but would not resonate; common exit tried, resonance good, and report said, "up to time pair exploded, the running seemed smoother, less noisy." |
| "(d) Dual or multiple engine arrangement" | (d) A form of cascading was tried having one jet exhaust in succeeding jets diffuser; only one of two would resonate at a time. |

Miscellaneous: Mr. Valerino mentioned a few other attempts of gadgetry. Ceramic tubes were used successfully. A series of concentric water spray rings at exhaust were tried to decrease noise but were unsuccessful; a bent tube was tried and resonance occurred.

Further information on valves shows that rotary valves (rotating about axis of the tube) were tried successfully as long as ram air was induced in intake to valves. A butterfly valve arrangement, rotating about an axis normal to axis of the tube was successful without ram air. Both arrangements required use of an electric synchronous motor. It was found that in mounting check valves in conical banks or series of conical banks it was possible to double the thrust for the same cross-sectional valve area.

Further work on combustion showed that the jet would function employing nearly any common fuel with negligible increase in thrust. Various artifices were employed to encourage knocking and the formation of aldehydes with little change in thrust. It is reported by his associates that one worker tried a mixture of nitroglycerine and gasoline.

The instrumentation employed to obtain average velocities, pressures and temperatures were slight modifications of common types. The effort to get accurate measurements as a function of time is still underway and still constitutes somewhat of a headache.

A rather comprehensive coverage of the theoretical work done on pulse jets in this country is given in the bibliography.

Purpose

In most analyses of the pulse jet there has been some doubt expressed as to the existence of a plane of discontinuity at the flame, residual gas interface. The prerequisite purpose of this experiment, as designed, is to verify the existence of this discontinuity by photographing it as it reflects at the open end of the tube, acting as a source of a spherical wave in space. If this discontinuity exists, or, if there is a wave of sufficiently rapid change in density at the interface to cause a divergence of light rays from a point source passing through this change in density, then the primary aim of this experiment may be realized, i.e., a study of the velocity and density of a pulse jet exhaust as a function of time.

Instrumentation and Equipment

The jet employed was a model aeroplane unit produced by Aero-marine Corporation. (Plate 9) Fuel tank used was a liter flask. The ignition circuit consisted of a 6 volt Hotshot battery, a Ford Model T vibrator, a spark plug and switch. (Plate 10) The photographic equipment was composed of a point source of light and film holders. The point source consisted of a transformer, a 705 A rectifying tube and its power supply, a .2 microfarad, 15000 volt condenser, and .5 millimeter platinum arcs (Plate 11) masked with a porcelain plug to act as a .5 millimeter diameter source of light.

The film employed was Kodak contrast process panchromatic and 35 mm x 100 foot film strip (Super XX).

The spark was triggered by means of a spring loaded plunger on which one of the platinum points was mounted. (Plates 11, 12)

Procedure

In the middle 1920's when talk was circulating around German air power circles about rocket propulsion and rocket augmentation, there appeared a dissenter, Paul Schmidt. Schmidt maintained that any effort to augment the already exploded rocket products with outside air would be fruitless in that the shock losses anticipated, and the resulting decrease in velocity would cancel the increased momentum the additional mass afforded.

He offered instead a piston-like transfer of energy from the expanding "energy" medium to the "momentum" medium as the "energy" medium expanded through a plane interface. The pulse jet tube was his means of obtaining such a transfer. He claimed the transformation could take place without shock losses and anticipated ratios of "momentum" mass to "energy" mass of ten to one or higher.

A rather nice, and up to now, popular method of regarding pulse jet phenomena is to consider the piston-like action of the "energy" on the "momentum" medium to be exactly that of a piston and that the waves which originate in front of an accelerated piston be present.¹ This conception lends itself handily to the employment of Riemann's treatment of waves of finite amplitude, as derived in Appendix I, for the first phases, (first several firings) which could throw light on the succeeding firings. The surfeit of waves potentially present in the running jet lends itself handily to nothing at all.

Two of the adherents to this way of thinking were Becket and Sauer⁷ who predicted a shock wave of .00003439" thickness, over which there would

be a pressure rise of $\frac{P_2}{P_1} = 2.195$ at the instant the plane interface reached the tube's end. This wave would be traveling at 1640 ft./sec.⁶ and would partially reflect at the tube end with a change of phase, at the same time creating a spherical wave in the atmosphere outside the tube's end. Though the initial character of the spherical wave would be that of the plane wave as far as pressure gradient, thickness, and velocity of propagation are concerned it would quickly deteriorate to a sound wave (in a matter of a few inches according to Riemann) with a velocity of propagation of the speed of sound in the local atmosphere.

This shock wave and its ultimate sound wave product are of such a nature as to be photographable by shadowgraphic means. Even if the shock actually entailed as little as one hundredth of the pressure rise indicated, its existence could be verified by the shadowgraph.

The equipment was designed to insure an opportunity for verifying the existence of these waves and to employ their position, should they exist, in facilitating the study of shadowgraph exhaust patterns as a function of time.

Light Source. The light source was designed to specifications recommended by Bellinger and Barnes of General Electric.¹³ It was calibrated by using a supersonic air jet, a film holder, and contrast process panchromatic 8" x 10" film plates. It was found that the light source to jet, jet to film ratios were not critical, the photographs of ratios, 2:1 to 10:1, tried giving both the standing shocks and the turbulent stream. (See Plate 16)

The spark gap was tried with and without the porcelain mask and, while both arrangements produced useable pictures, the masked spark

produced the clearer detail of the two.

Efforts were made to photograph the jet next using an 8" x 10" film holder held by an agile co-worker whose function was to remove the slide for the spark's flash and replace it before the light from the glowing jet tube could over-expose the film. It was found that good pictures could be made at a light-jet, jet-film ratio of 7 to 1 if the total exposure to the jet light did not exceed three seconds, a certain amount of prefogging prior to the spark being beneficial, seeming to increase the sensitivity of the film. Light-jet, jet-film ratios up to and greater than 10 were tried but fogging from the jet tube became prohibitive at ratios greater than ten with the amount of human delay existent in the calibration arrangements.

Film Boxes. A stand was built to accommodate the 8" x 10" film holder. A film exposure box (Plate 13) was built to hold 100 feet of 35 mm film to allow 5 feet strip exposures at a time. This was mounted on top of the film plate, overlapping by approximately 5 inches and so situated that the film in both boxes would be in the same plane.

It was found that the film in both boxes received sufficient light for recording a change in density regardless of whether the light source was placed facing the center or facing the ends. To reduce the time of exposure and insure an approximately uniform exposure duration to all film each run, a wedge shaped shutter was constructed and mounted on a pivot beyond the film strip. This shutter allowed the total exposure time to be cut to less than a second, and, since some fogging was desired, this was deemed adequate.

The Jet. The jet was mounted by a stainless steel clamp to a metal

bench on which was placed the ignition system and fuel tank.

The fuel tank was a glass flask, mounted on a ring stand. The fuel line to engine and an air vent were inserted in the top of the flask. The fuel level was kept between 5/8" and 1" below the center of the jet; this level was not critical in running, but was in starting. White unleaded Amoco gas was used as fuel. The ignition circuit consisted of a 6 volt Hotshot battery, a Ford Model-T coil and a spark plug.

To start the engine a small stream of compressed air was directed normal to the fuel feed orifices such that the air fuel mixture was aimed at the spark plug on the inside of the tube. Engine warm-up time varied from 1 to 5 seconds before the jet would start resonating. As soon as the engine was resonating the compressed air stream was removed and the ignition switch cut off. After eight seconds running time the jet tube temperature and the fundamental resonating frequency became stable at which time pictures were made. Preliminary fundamental frequency measurements were made by comparing the tone of the tube with a taut wire which had been calibrated and was found to be 240 pps.

Operation. In photographing spherical shock or sound waves the light from the point light source passes through a point of tangency to the sphere. If there is an abrupt change in the rate of change of density along the direction of propagation such that $\frac{\partial^2 \rho}{\partial x^2} > 0$ there is a divergence of the light passing through the change and such a divergence would record on the film as a shadow.

In the case of rarefaction waves where $\frac{\partial^2 \rho}{\partial x^2} < 0$ the light would converge and the convergence would record on the film as a heavily exposed line or bright line.

The curve described by the points of tangency of the line of light on successive spheres radiating from the jet end is a circle with center midway between the point source of light and the source of sound with radius half the distance from the light to the tube (Appendix II).

Several efforts were made to record this point of tangency.

Attempts at Recording Shock Front. The spark gap was placed on a plane with and 112" from the axis of the jet tube. The 8" x 10" film plate was placed 16" beyond the tube's axis and perpendicular to this plane. On top of the box was placed the five-foot strip, overlapping by approximately 5 inches the plate, giving a total length of film of 5'-7".

This length of film so placed would record any spherical wave front emanating from the tube's exhaust up to 4.6' in radius. Since the jet was firing at 240 cycles per second, each firing permitting the birth of a new wave if such a wave existed, these waves, if traveling at the speed of sound at the local atmospheric temperatures would be 4.57' apart, or for each photograph there would be at least one wave front on the film.

Fifteen pictures were taken at random intervals using this geometry.

The exhaust patterns photographed recorded on the 8" x 10" plates showed such a thorough coverage of the flow forms expected for the various portions of the cycle that a wave velocity as much as five times that of sound could not have escaped perception should it have existed. Though the turbulent flow pattern of the jet exhaust was recorded on all the film strips, no abrupt change in density in the form of a condensation wave was recognized.

There seemed a possibility that the sharp divergence could have

occurred in the shock front but could have been so distorted by refractions occurring between the point of tangency to the shock sphere and the film, that it would be rendered unrecognizable.

To remedy this possible failing the 5' film box was next mounted on an adjustable stand, its midpoint placed slightly behind the curve of intersection of the light and the successive shock spheres, and parallel to a tangent of this curve at this point. This arrangement facilitated a coverage of waves with radii up to 4.75' from the tube's end.

The film box was furnished with a spring loaded shutter; the 8" x 10" film plate was retained in its original position.

Again fifteen runs were made, again with negative results. Using this geometry the only turbulent flow patterns recorded were on the forward end of the film strip. It is not thought likely that this turbulence could have distorted a possible shock front beyond recognition for the following reason: The sought shock front should precede the exhaust or herald it; the exhaust gases exit with sufficient momentum as to be well beyond the tube before a new cycle starts; it is only at a distance of three or four feet from the tube's end where damping has taken place to such an extent that the new shock front could have caught up with the preceding exhaust's gases.

The final attempt was designed to record the waves in the event they should assume the forms of those originating from a firearm's muzzle blast. Measurements have been made¹¹ showing the waves to be propagated at the speed of sound but from a source that moves forward from the muzzle. These same waves are complete spheres with recordable pressure rises 180° from the direction of firing.

By placing the film strip behind the curve of tangents in the direction opposite that of the exhaust, any sphere's tangent may be recorded, the higher the initial velocity of propagation, the closer these waves will appear.

In this arrangement as in the preceding arrangement the maximum radius of the recordable sphere was 4.72'.

In this case, however, since the speed of sound is the maximum velocity in which the wave may be propagated in the direction of the film strip, we are assured of at least one wave appearing for each photograph taken. One photograph was taken; the results were again negative.

Results

The quest for the wave of finite amplitude was negative.

Without this wave and its capacity to indicate the portion of the cycle recorded it is impossible to make an accurate analysis of the exhaust shadowgraphs as a function of time.

We may, however, with free use of the imagination make a few surmises concerning these shadowgraphs with the results obtained.

It seems reasonable to assume that the initial and final portions of the exhaust flow will be mushroom shaped, or more vividly, two mushrooms, one large, the other small, placed end to end, the larger being the head of the initial exhaust, the smaller being the tail of the exhaust flow. With this assumption we may employ some of the shadowgraphs taken in quest of the shock front to give an approximate analysis of the flow cycle.

Figure 4 gives the flow patterns observed from seven representative

film strips taken with the film strip 5" above and 16" to the rear of, and parallel to the jet exhaust.

In Appendix II corrections were made from the geometry of the photographic plane. Plates 1 through 7 show successive phases of the cycle and may be compared with the corrected mushroom patterns in Figure 5.

Assuming the position of the mushroom first appearing on Strip 2, Figure 5, to be $18\frac{1}{2}$ " away from the jet barrel at the time of exposure and at the end of the exhaust portion of the cycle, (Strip 5) Figure 5, the head of the same mushroom is $36\frac{1}{2}$ " from the exhaust. Since the exhaust constitutes sixty per cent of a cycle of duration .00416 second, the time for this wave to travel this distance is .0025 of a second. Thus the average velocity of an ejected quantity of air over the region 1.5 to 3 feet from the jet exhaust may be approximated as 600 feet per second.

Plates 6 and 7 showed standing rarefaction waves at the lip of the jet tube. (Plates 14 and 15 are enlargements of Plates 6 and 7, respectively, showing these rarefaction waves.)

In Plate 14 the flow into the tube has just begun the position of the rarefaction wave indicating that the air flowing into the tube is unable to turn the sharp corner without separation taking place at that point.

In Plate 15 the rarefaction wave has moved around the lip of the tube showing that now separation takes place at even an earlier instant.

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APPENDIX I

DERIVATION OF RIEMANN RELATIONSHIPS FOR WAVES OF FINITE AMPLITUDE

Assume one dimensional movement along the axis of a tube. Consider a pressure wave in a tube of constant cross-section, initiated by a perturbation at $X = t = 0$ travelling to the right at constant velocity, a . Let the density be ρ in front of the wave, and the velocity of the air, $u = 0$. Behind the wave the density has undergone a change $d\rho$ and is now $\rho + d\rho$ whereas the velocity is now du . Assume the point of view of riding on the wave front. The gas in front of the wave enters at a velocity $= -a$ and a density ρ ; while the air leaves at a velocity $= -a + du$ and a density $\rho + d\rho$. Since there is no change in cross-section $-\rho a = (\rho + d\rho)(-a + du)$ or with a sufficient degree of accuracy $du = -\frac{1}{\rho} a d\rho$. Assuming an isentropic compression wave,

$$a = \sqrt{\frac{dp}{d\rho}}$$

$$= \sqrt{k \frac{p}{\rho}}$$

therefore

$$u = - \int_{\rho_0}^{\rho} \frac{1}{\rho} \sqrt{k \frac{p}{\rho}} d\rho$$

or

$$u = - \frac{2\sqrt{k \frac{p_0}{\rho_0}}}{K-1} \left(\rho^{\frac{K-1}{2}} - \rho_0^{\frac{K-1}{2}} \right)$$

or

$$u = -\frac{2}{k-1} (a - a_0)$$

Similarly for an inward flowing wave,

$$u = \frac{2}{k-1} (a - a_0)$$

or

$$\Delta u = \mp \frac{2}{k-1} \Delta a$$

(1)

From (1) Riemann defined the terms,

$$r = \frac{1}{2} \left(\frac{2}{k-1} a + u \right)$$

$$s = \frac{1}{2} \left(\frac{2}{k-1} a - u \right)$$

(2)

such that for each outwardly traveling wave there belongs an r value that does not change even when an inwardly traveling wave is encountered and an s value that is constant if an inwardly traveling wave is not encountered.

Similarly there belongs to each inwardly traveling wave an s value which does not change if an outwardly traveling wave is encountered and an r value which is constant if an outwardly traveling is not encountered.

A more general derivation may be obtained directly from Euler's equation of motion and the equation of continuity:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = -\frac{1}{\rho} \frac{\partial p}{\partial x}$$

and

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} = -\rho \frac{\partial u}{\partial x}$$

for one dimensional flow.

Giving w the meaning

$$w = \int_{p_0}^p \frac{a}{\rho} dp$$

or

$$\frac{\partial w}{\partial t} = \frac{a}{\rho} \frac{\partial \rho}{\partial t} \quad \text{and} \quad \frac{\partial w}{\partial x} = \frac{a}{\rho} \frac{\partial \rho}{\partial x}$$

Substituting above the equation of motion becomes

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = -a \frac{\partial w}{\partial x}$$

and the equation of continuity

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} = -a \frac{\partial u}{\partial x}$$

Adding and subtracting we get

$$\left(\frac{\partial}{\partial t} + (u+a) \frac{\partial}{\partial x} \right) (w+u) = 0$$

$$\left(\frac{\partial}{\partial t} + (u-a) \frac{\partial}{\partial x} \right) (w-u) = 0$$

which imply that $w+u = \text{constant}$ for a particle moving with a velocity $a+u$, $w-u$ is a constant for a particle moving with a velocity of $u-a$.¹⁰

A detailed explanation of the method of application of the above relationships may be found in Reference 1.

APPENDIX II

Geometry of the Photographic Plane

Figure 2 shows any sound circle with source at $(b,0)$ and its tangent light ray in the plane of the jet axis and the light source.

The point Q has coordinate x,y related such that $y = x \tan \phi$.
 $\tan \phi = \frac{b-x}{y}$, therefore $y^2 + x^2 = bx$ is the locus of intersections with which we are concerned. This is a circle of radius $\frac{b}{2}$ with center at $x = \frac{b}{2}$, $y = 0$.

Since the spheres and their tangent rays are symmetrical, we may revolve $x^2 + y^2 = bx$ about x and get the equation of the sphere which is the locus of all points in space where a ray of light at the origin is tangent to a sphere of any radius with center at $x = b$, $y = 0$, namely $x^2 + y^2 + z^2 = bx$, where z is an axis perpendicular to the x,y axes.

Figure 3 shows the sphere of intersection and the arrangement of the photographic plates.

For the position (1) the distance of the spherical wave to the tube source may be calculated thus:

The wave would be recorded on the strip as a line approximately parallel to the z axis, 1 distance from the x axis. The equation of the line from the light source at the origin to point $(b+c, 1, d)$ is

$$y = \frac{1}{b+c} x, \quad z = \frac{d}{b+c} x$$

Substituting these in the equation for the sphere and solving for x and y we get

$$x = \frac{b}{1 + \frac{l^2}{(b+l)^2} + \frac{d^2}{(b+l)^2}}$$

and

$$y = \frac{l}{b+l} \frac{b}{1 + \frac{l^2}{(b+l)^2} + \frac{d^2}{(b+l)^2}}$$

$$z = \frac{d}{b+l} \frac{b}{1 + \frac{l^2}{(b+l)^2} + \frac{d^2}{(b+l)^2}}$$

The radius of the sphere then is

$$r = \sqrt{(x-b)^2 + y^2 + z^2}$$

which when divided by the local speed of sound gives the time elapsed from the waves emergence until the time of the flash.

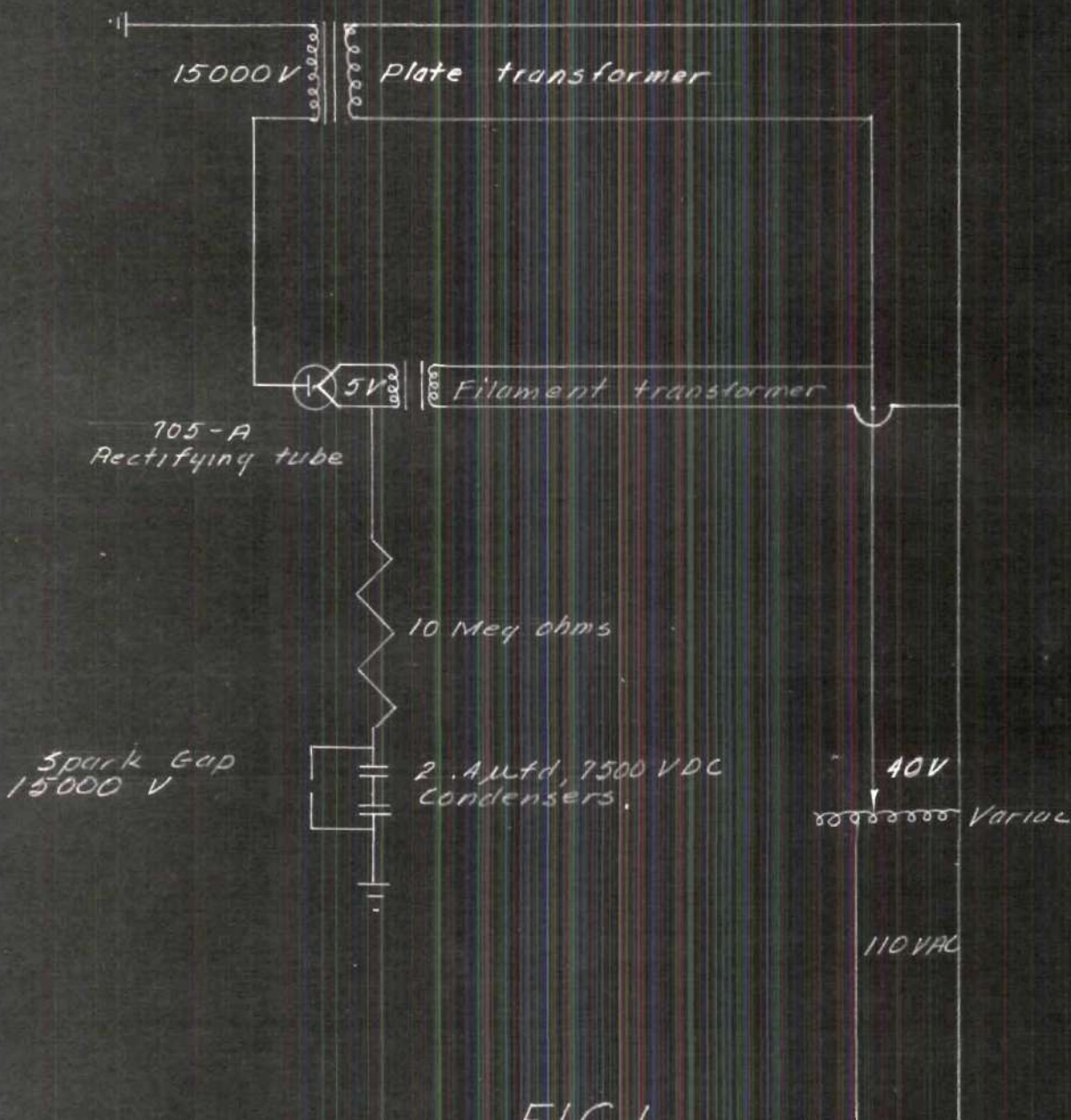
Constants are: $b = 112''$, $c = 16''$, $d = .6''$.

Similar calculations may be made for positions (2) and (3).

To obtain the actual locations of the mushroom flow patterns similar triangles were used.

APPENDIX III

FIGURES



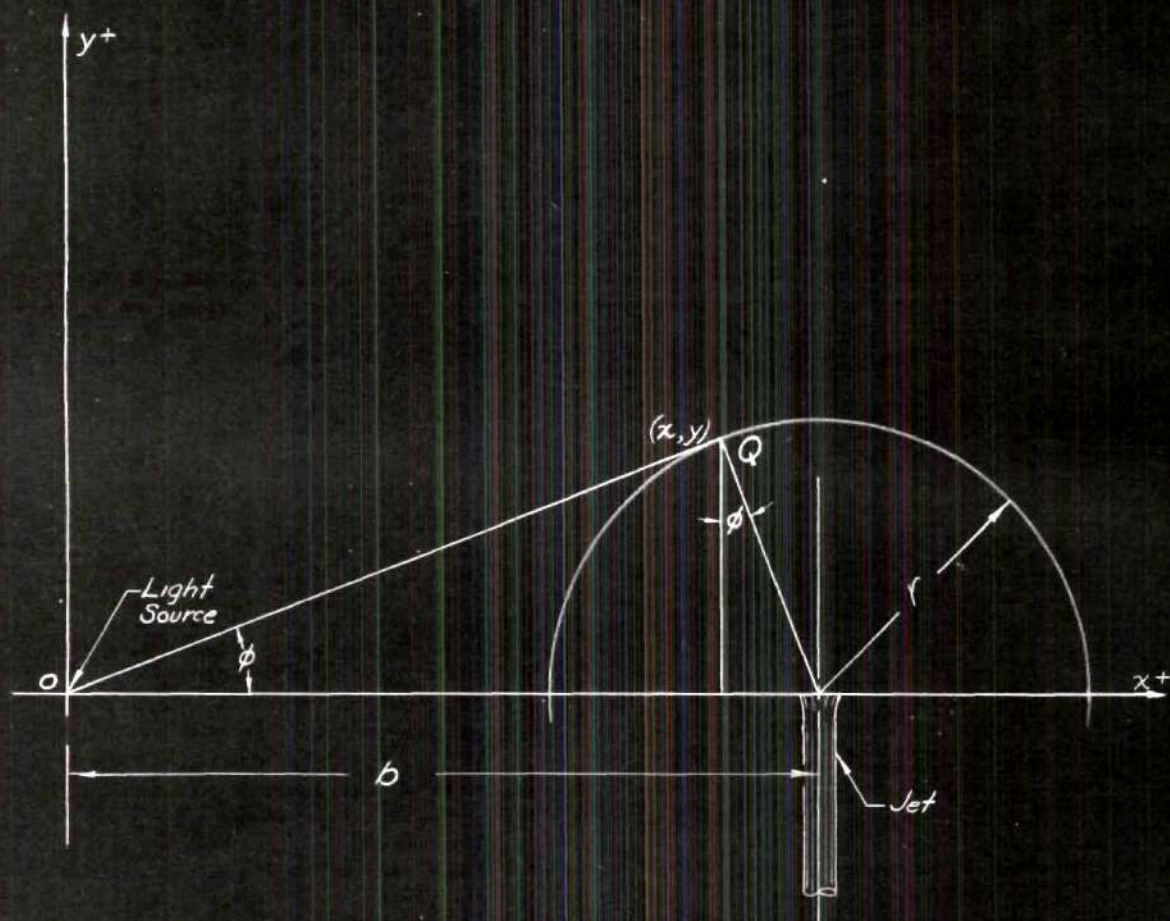


FIG. 2

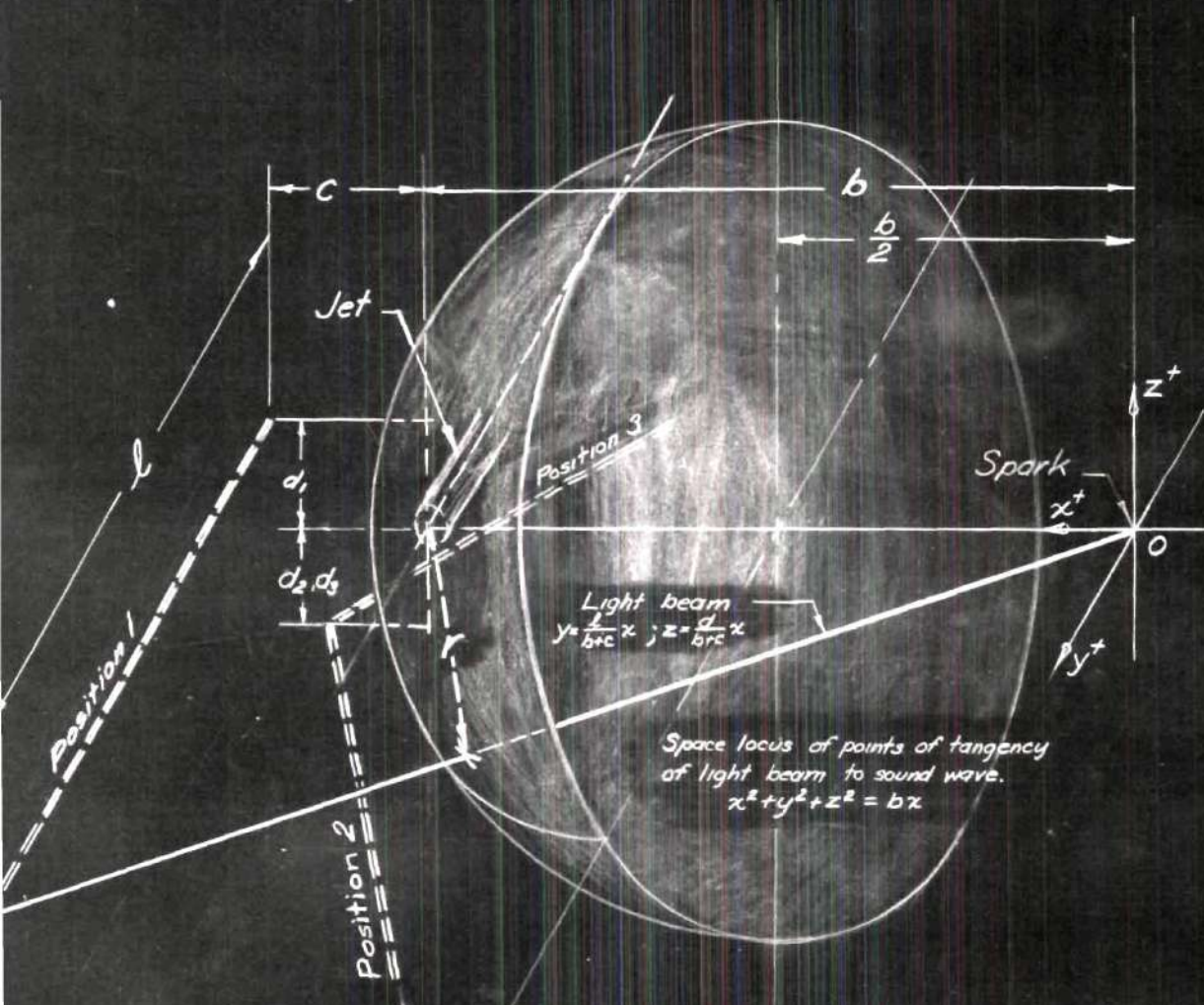


FIG. 3

PLATES

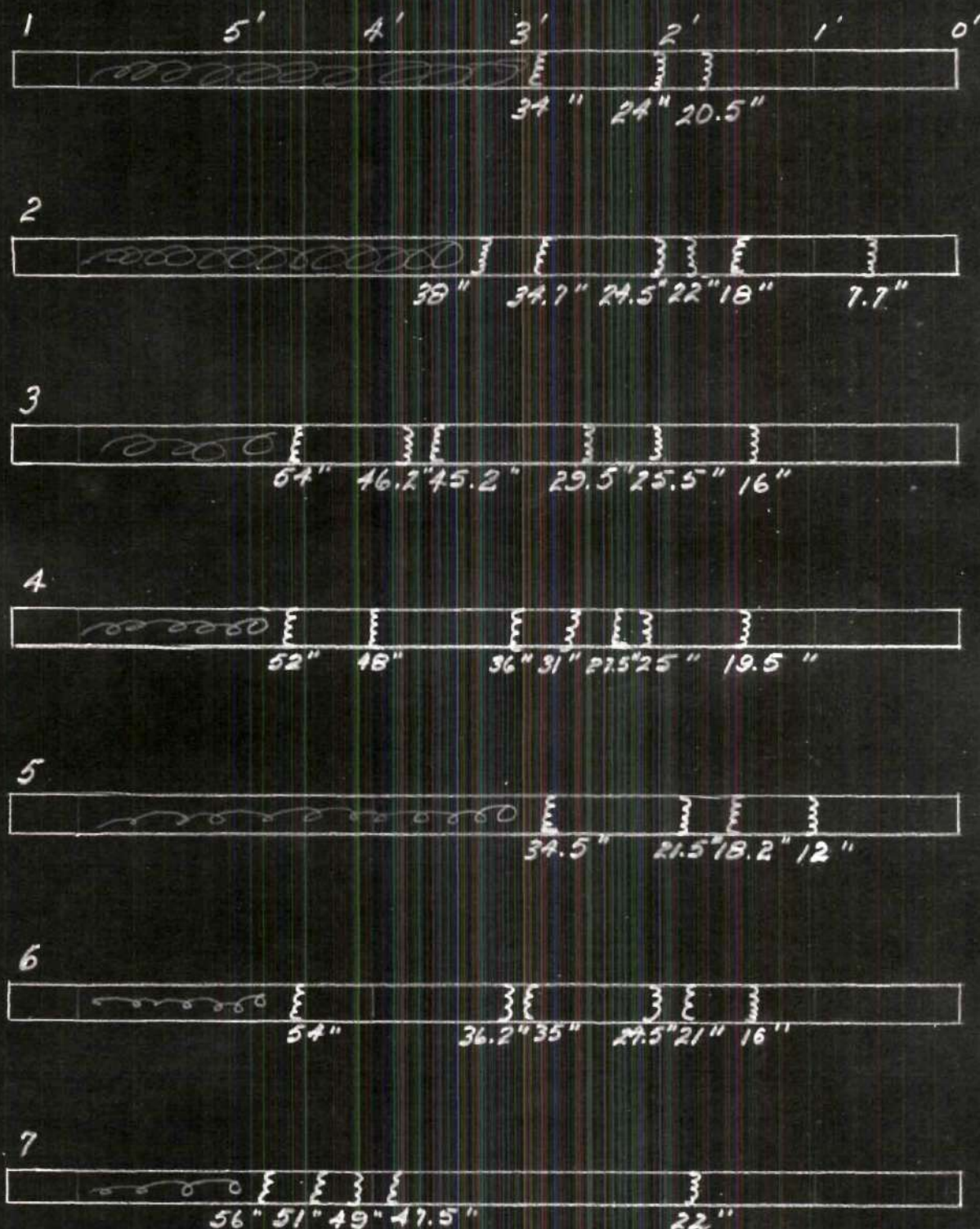


FIG 4 EXHAUST PATTERNS
AS RECORDED POSITION 1.

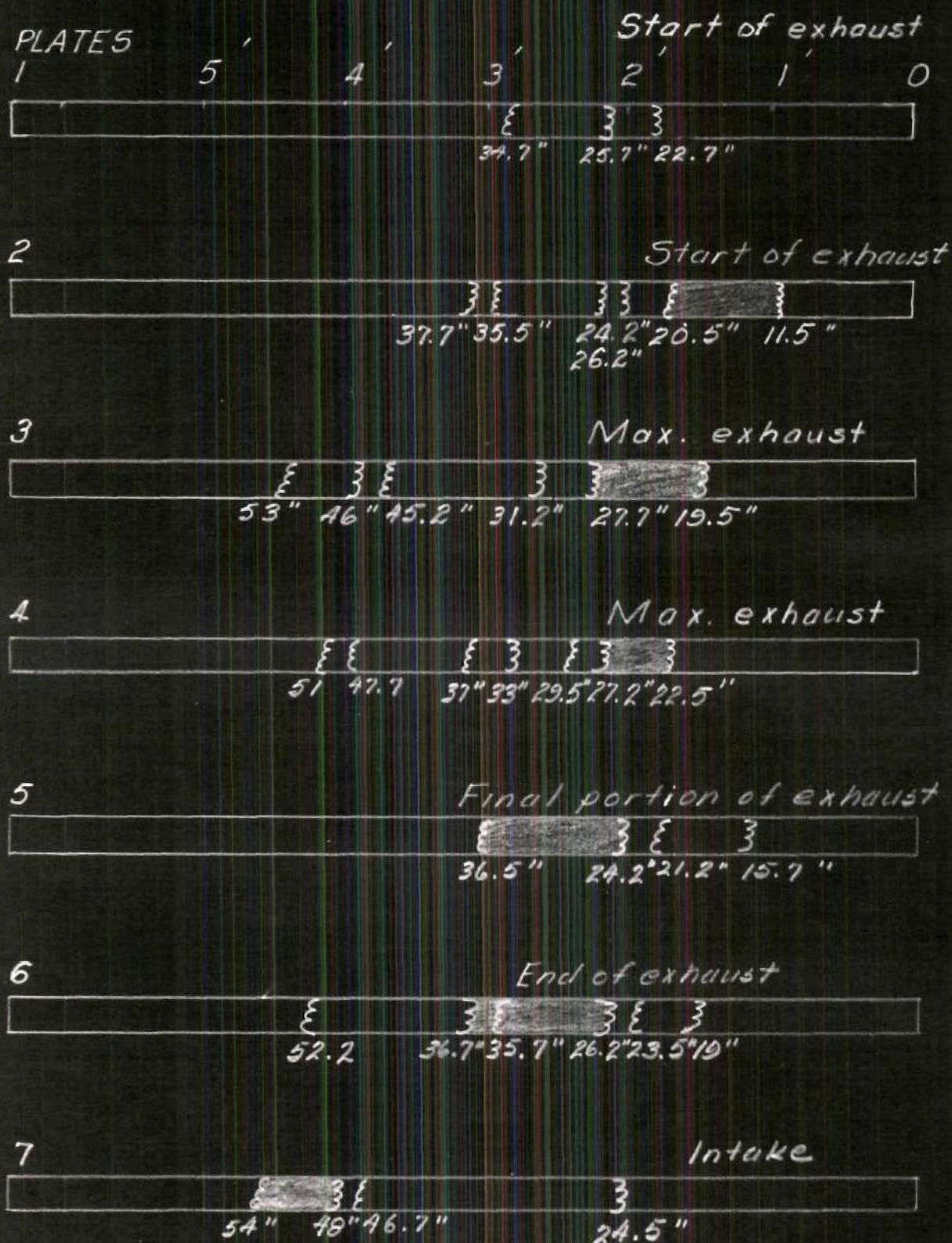
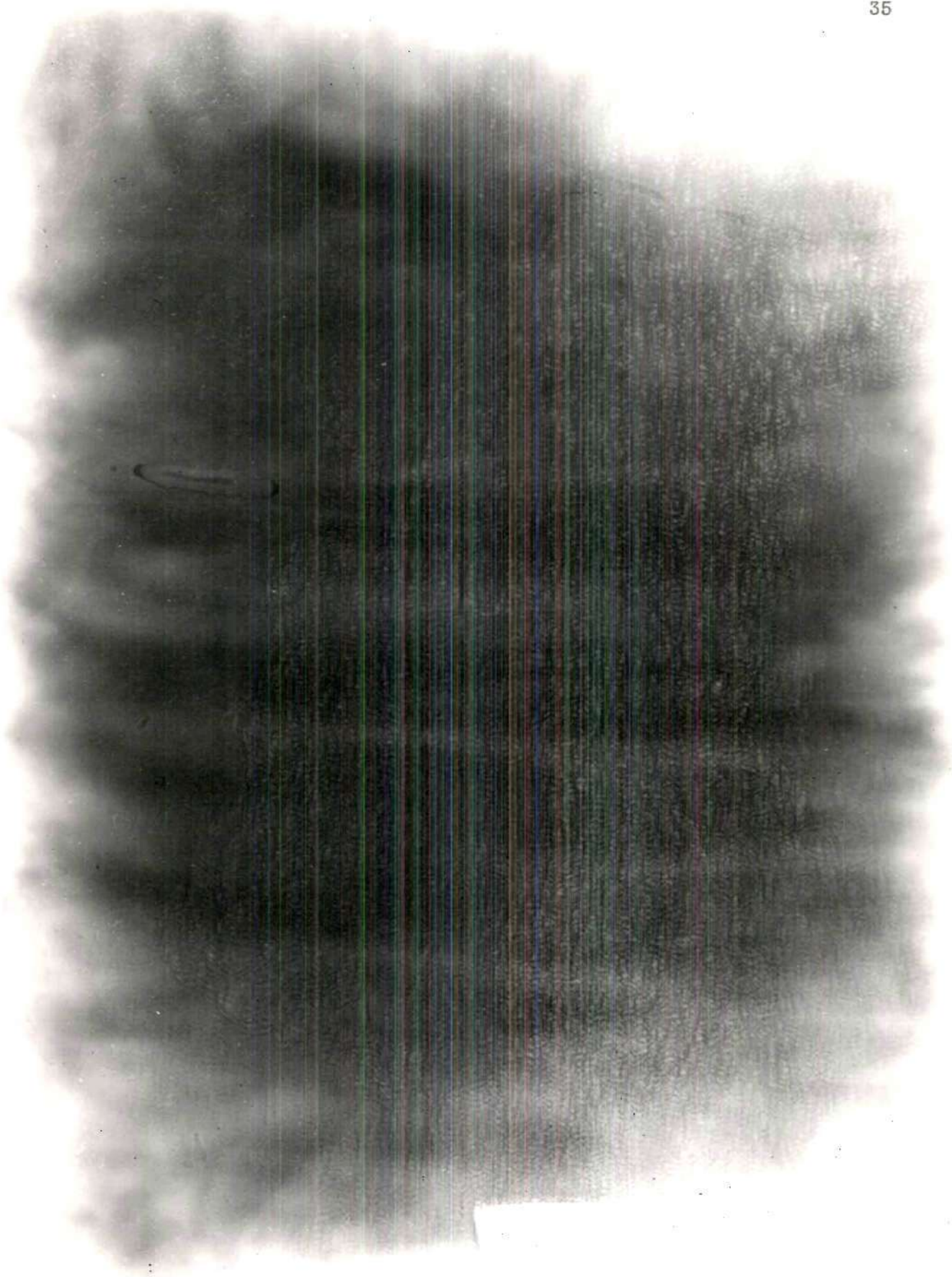
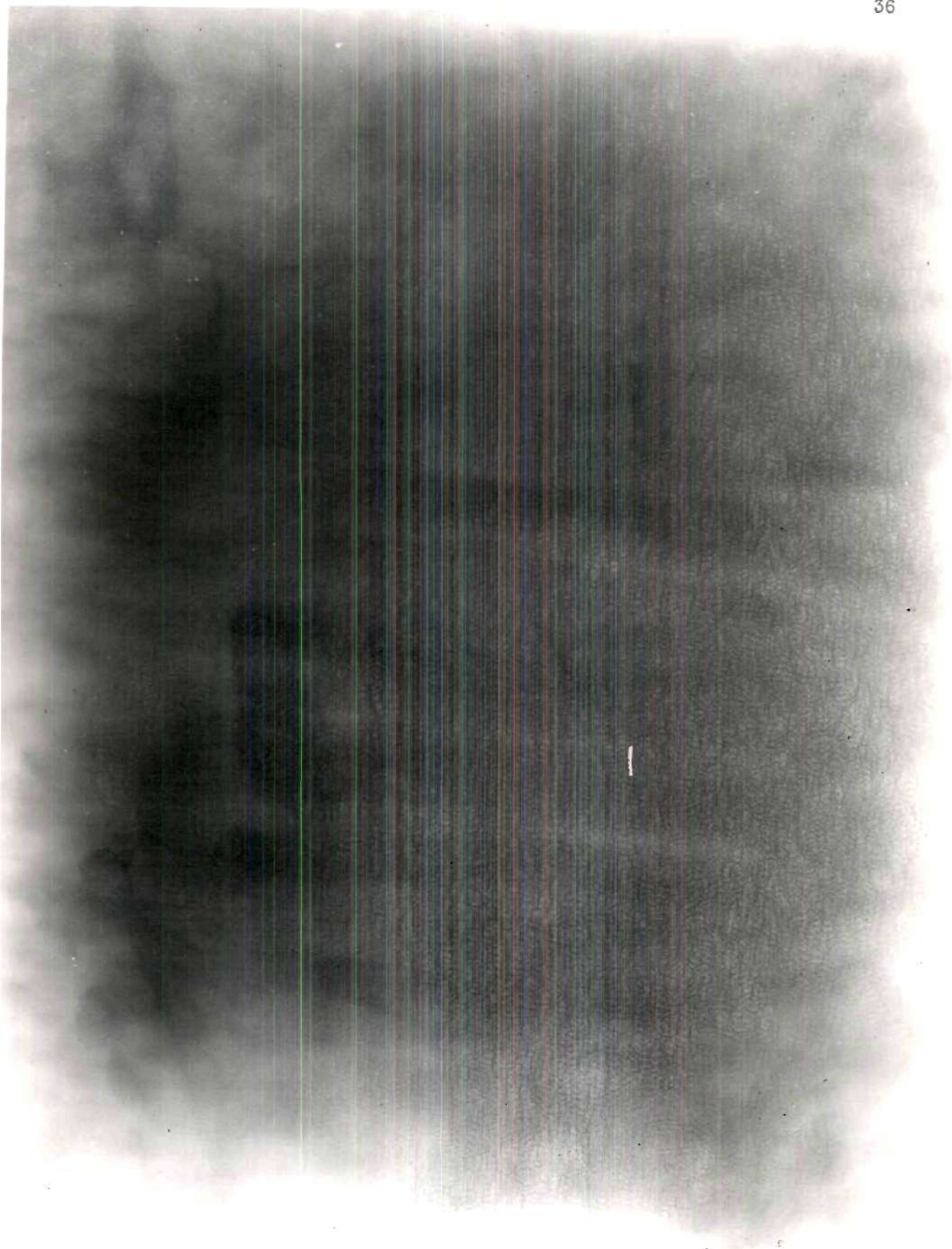


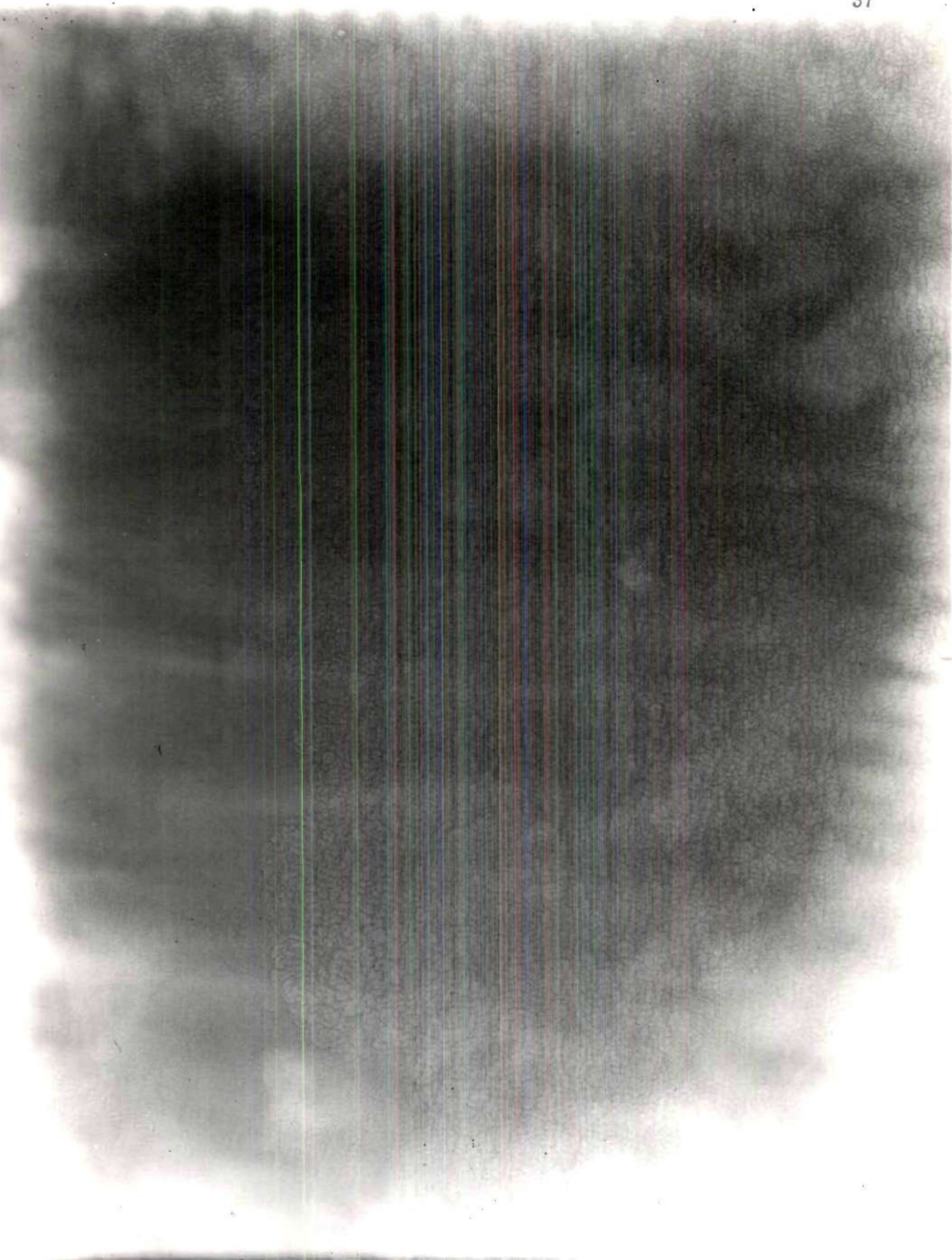
FIG 5. ACTUAL EXHAUST PATTERNS CORRECTED AND LABELED

APPENDIX IV

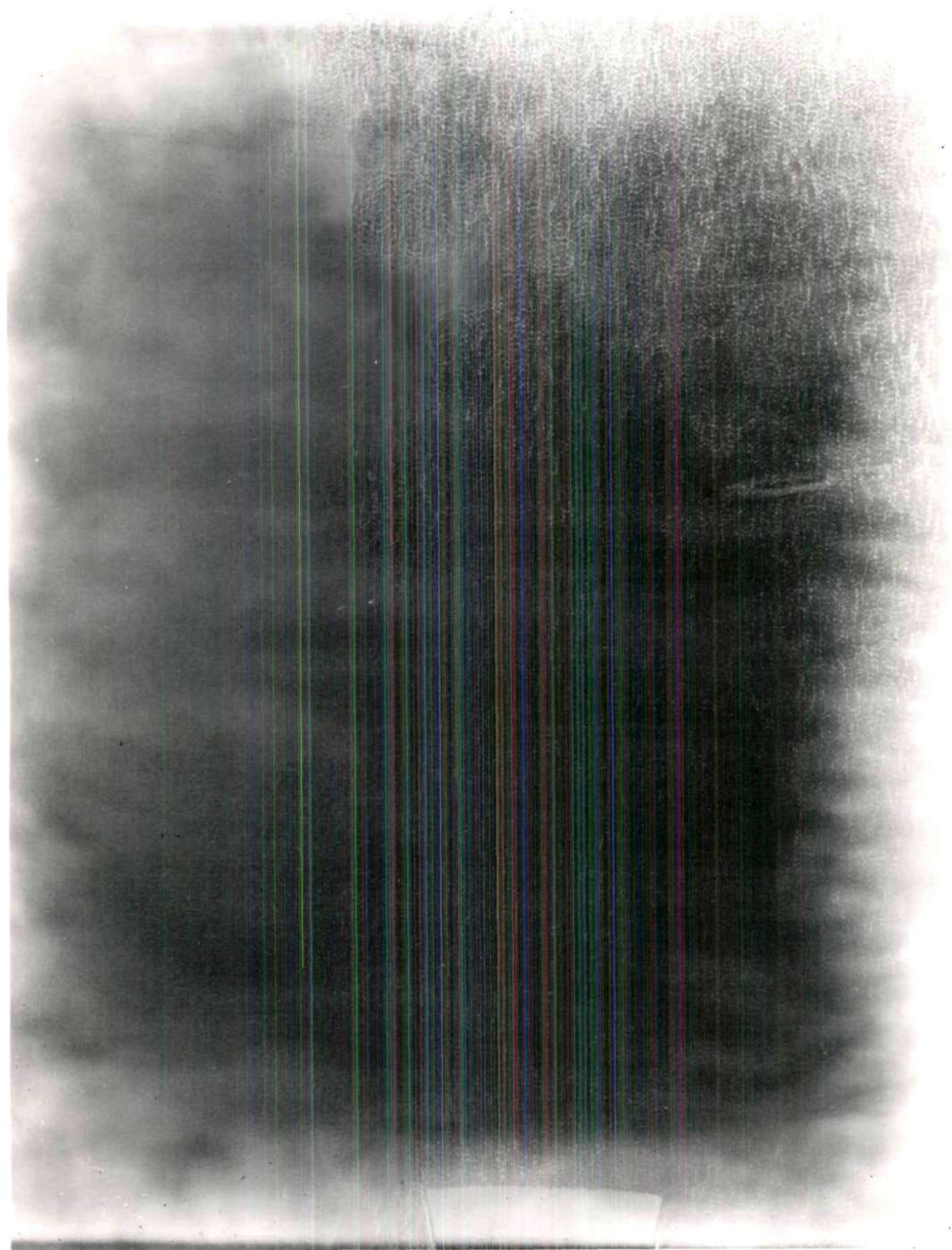
PHOTOGRAPHS AND SHADOWGRAPHS

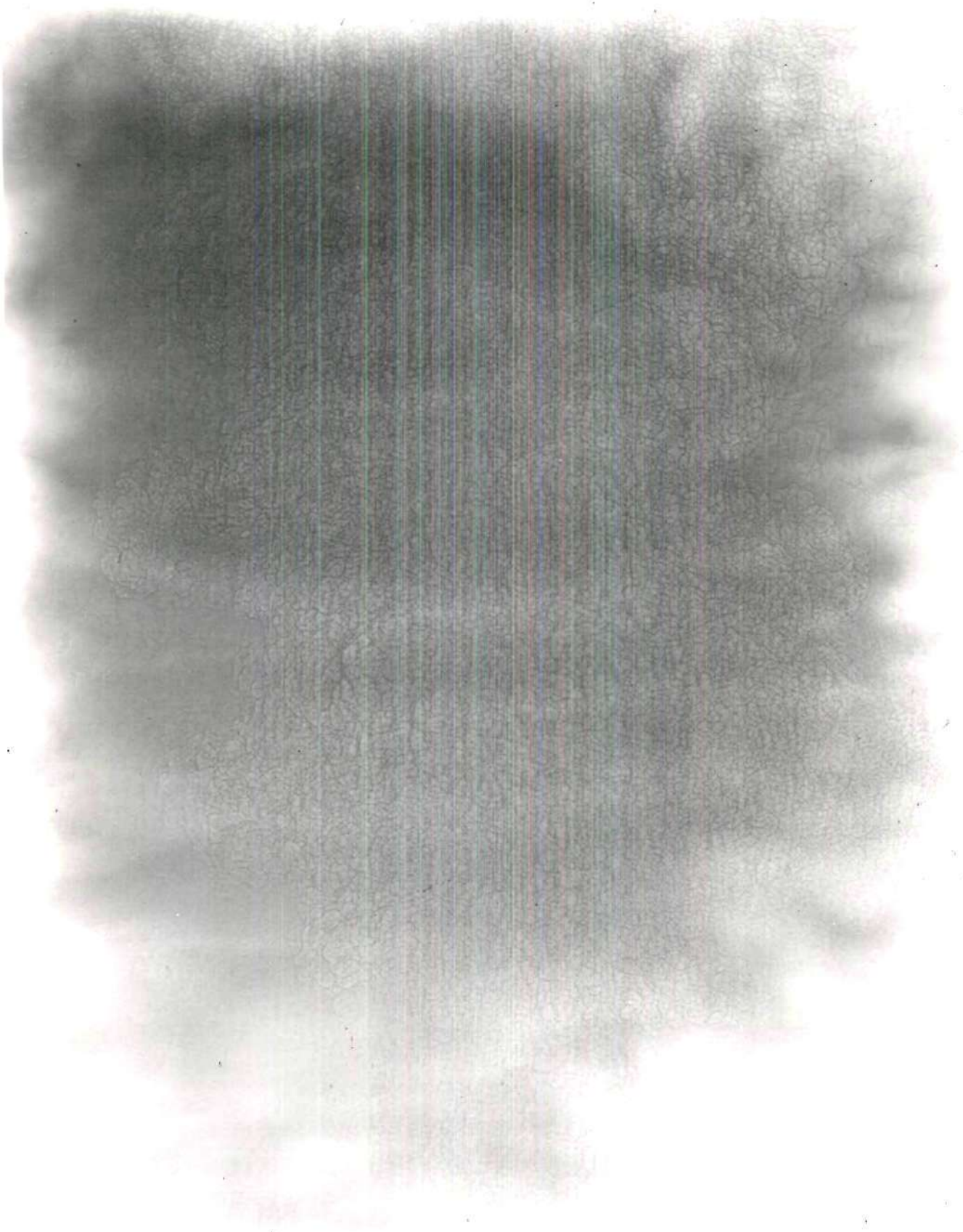


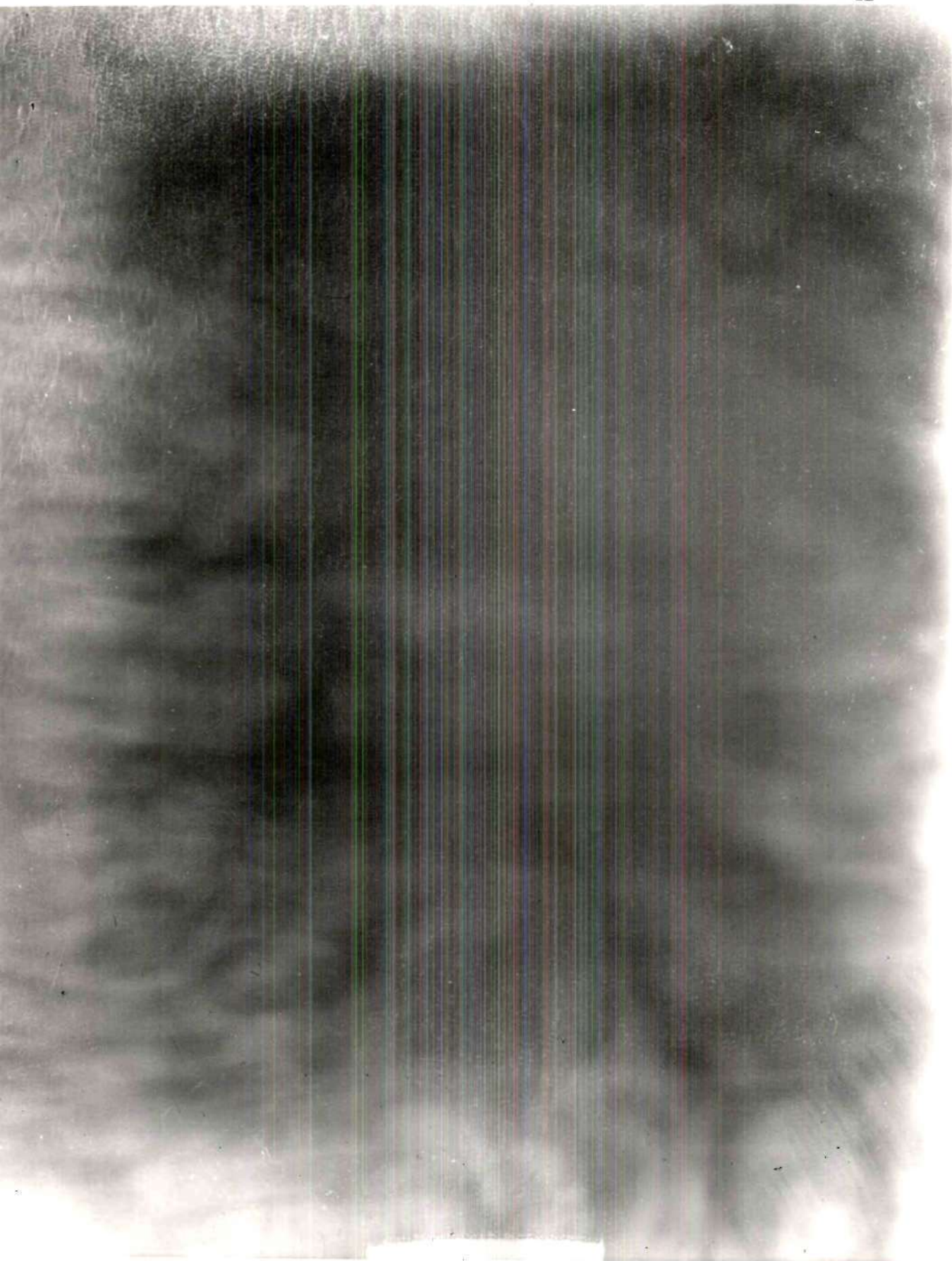












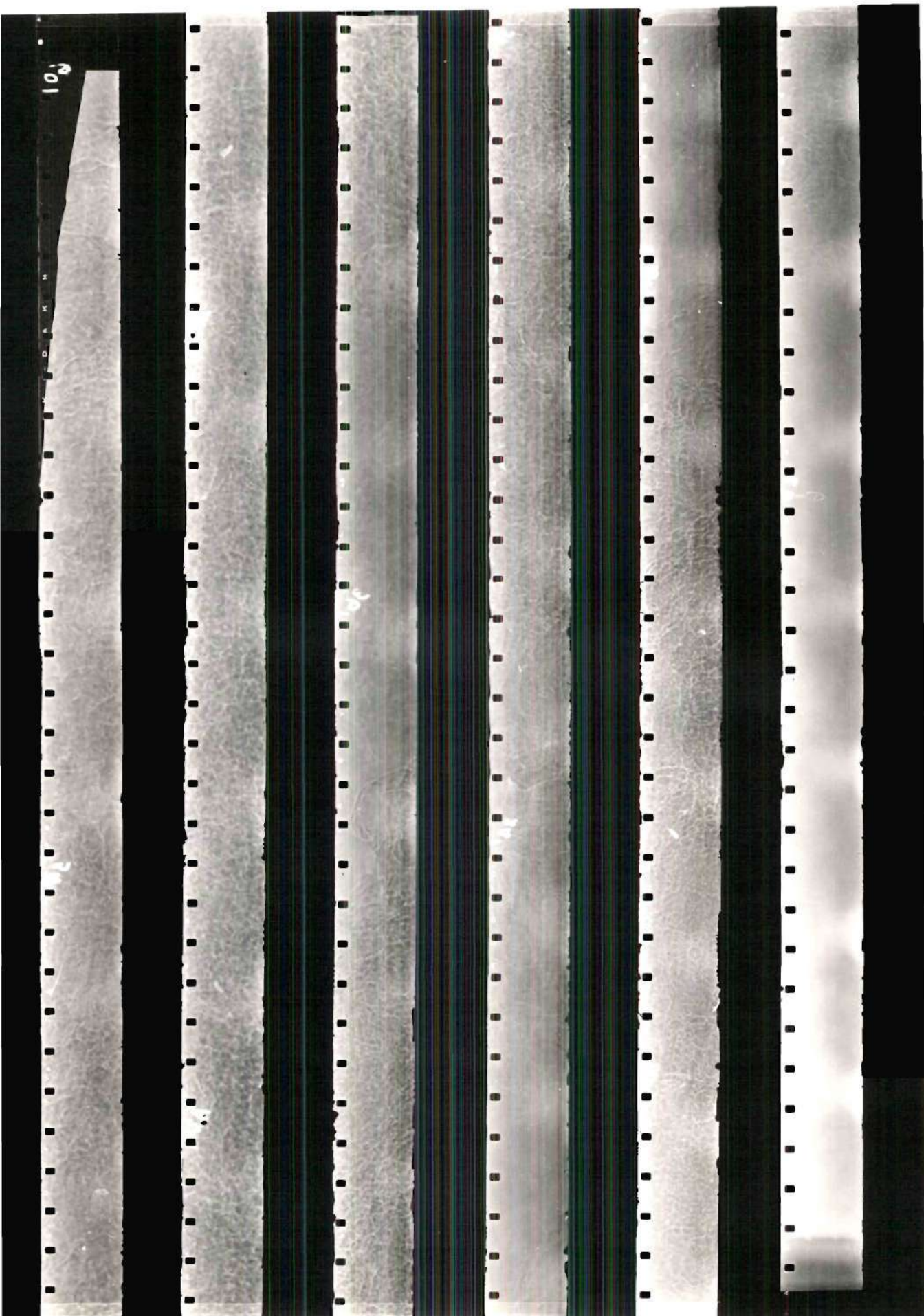




Plate 9

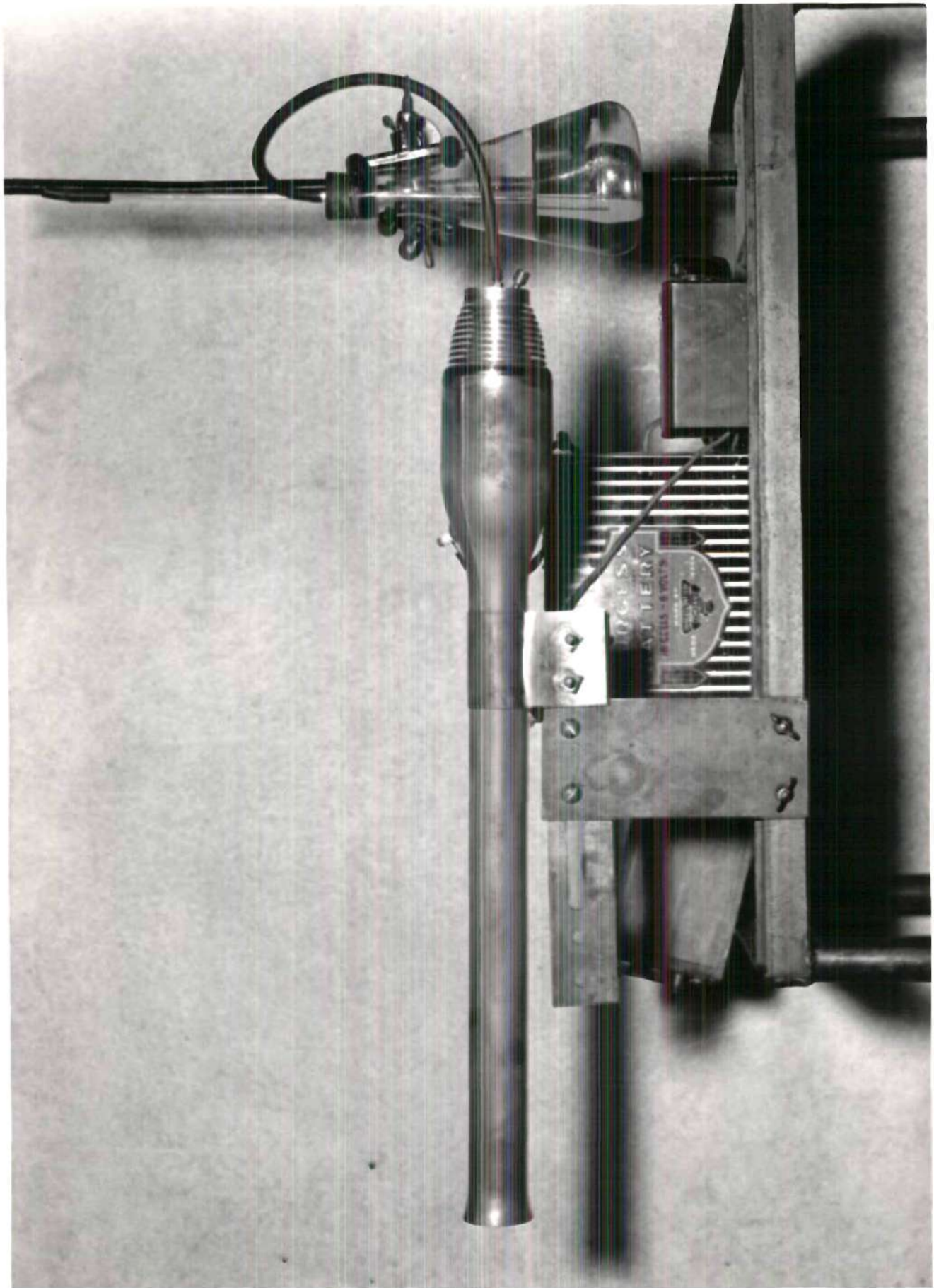


Plate 10

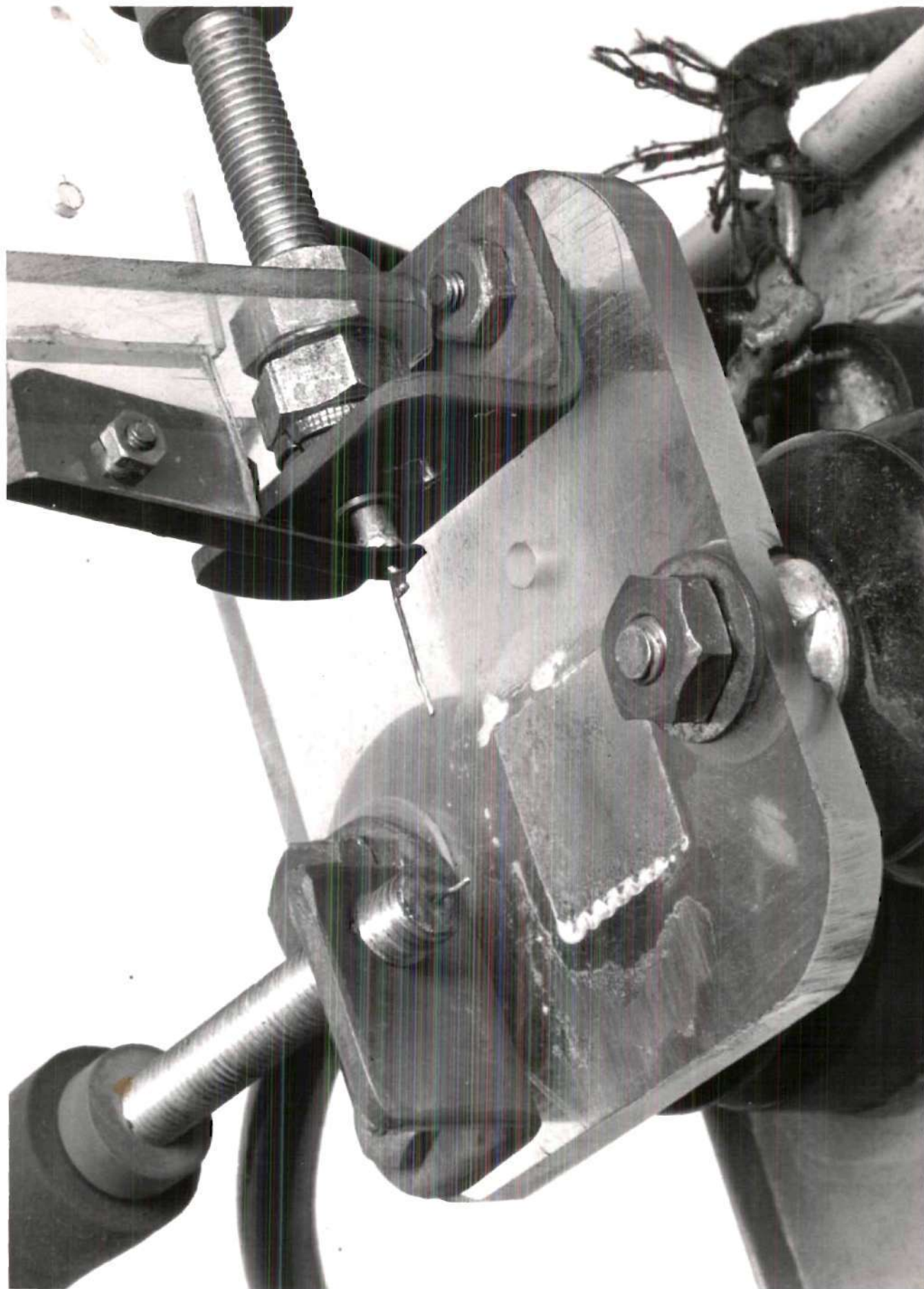


Plate 11

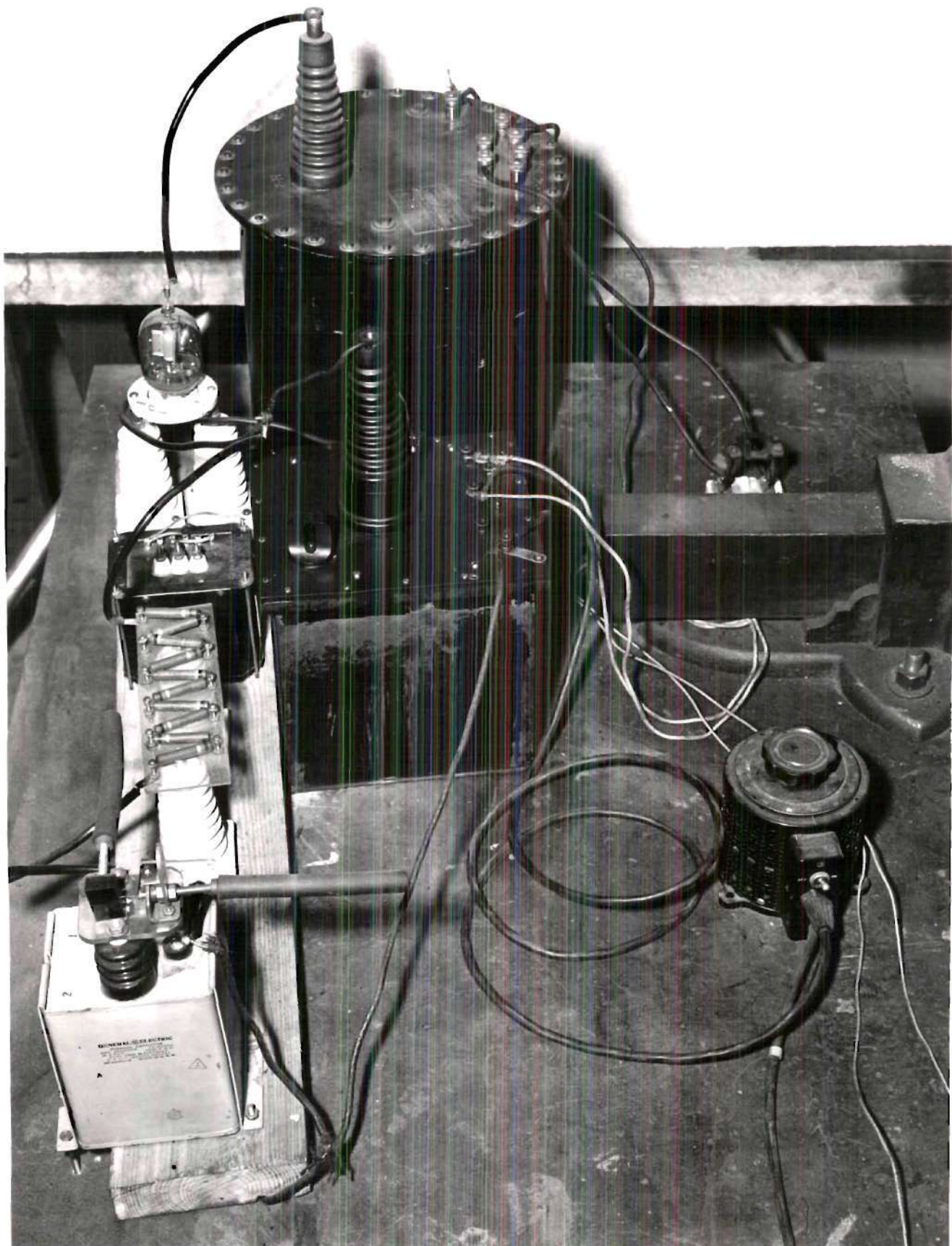
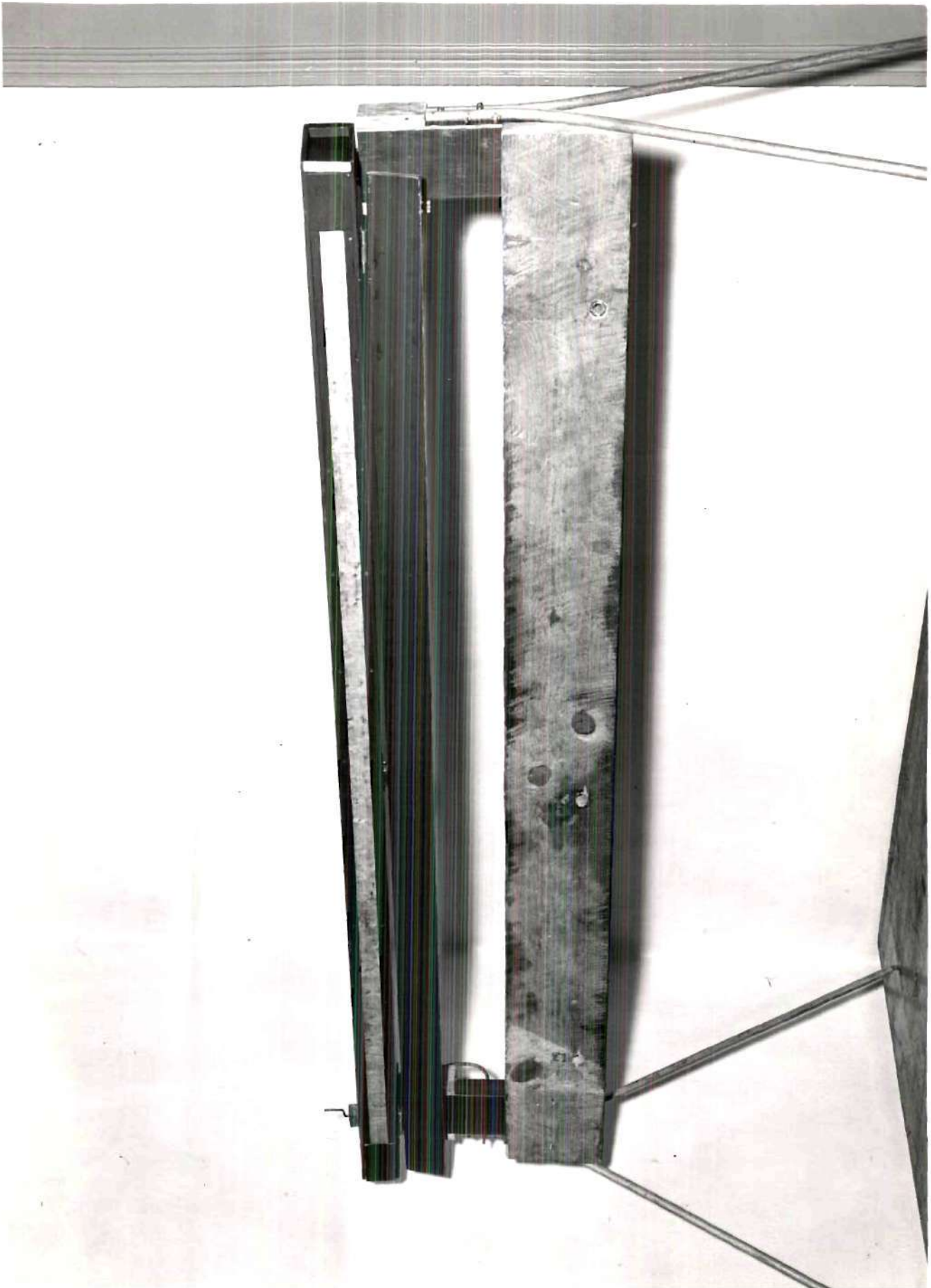


Plate 12





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